

## DYNAMICAL MASS SEGREGATION ON A VERY SHORT TIMESCALE

RICHARD J. ALLISON,<sup>1</sup> SIMON P. GOODWIN,<sup>1</sup> RICHARD J. PARKER,<sup>1</sup> RICHARD DE GRIJS,<sup>1,2</sup>  
SIMON F. PORTEGIES ZWART,<sup>3</sup> M. B. N. KOUWENHOVEN<sup>1</sup>*Draft version June 26, 2009*

## ABSTRACT

We discuss the observations and theory of star cluster formation to argue that clusters form dynamically cool (subvirial) and with substructure. We then perform an ensemble of simulations of cool, clumpy (fractal) clusters and show that they often dynamically mass segregate on timescales far shorter than expected from simple models. The mass segregation comes about through the production of a short-lived, but very dense core. This shows that in clusters like the Orion Nebula Cluster the stars  $\geq 4M_{\odot}$  can dynamically mass segregate within the current age of the cluster. Therefore, the observed mass segregation in apparently dynamically young clusters need not be primordial, but could be the result of rapid and violent early dynamical evolution.

*Subject headings:* methods: N-body simulations — stars: formation — stellar dynamics — galaxies: star clusters

## 1. INTRODUCTION

Most stars are observed to form in clusters (probably 70–90%; Lada & Lada 2003.) Therefore, understanding cluster formation is the key to understanding most star formation. Unfortunately, observations of very young clusters ( $\ll 1$  Myr) are hampered by extinction due to those clusters still being embedded in their natal molecular clouds. Most often we are restricted to observing older clusters which is problematic as these clusters may well have undergone significant dynamical evolution. In particular, young clusters seem to expand significantly in the first few Myr (Bastian et al. 2008), possibly due to the effects of multiple stellar encounters (van den Berk et al. 2007) or by the rapid expulsion of primordial gas (Goodwin & Bastian 2006).

A potentially significant observation is that many young clusters appear ‘mass segregated,’ i.e., the most massive stars are concentrated towards the center of the cluster (Hillenbrand & Hartmann 1998; de Grijs et al. 2002a,b,c; Gouliermis et al. 2004). Clearly, mass segregation could be primordial – clusters may form with the most massive stars concentrated at or near the center. Alternatively, mass segregation could be dynamical – the most massive stars migrate into the center of the cluster after formation due to two-body interactions. As well as potentially providing constraints on clustered star formation, the origin of mass segregation may help distinguish models of massive star formation. In particular, are the masses of the most massive stars set by the mass of the core in which they form (e.g., Krumholz et al. 2007), or by competitively accreting mass due to a favorable position in the cluster (e.g., Bonnell et al. 1998; see also Krumholz et al. 2005, and Bonnell & Bate 2006)? If mass segregation is primordial it might well argue in favor of competitive accretion, as in that scenario massive stars should form in the center of a cluster.

Bonnell & Davies (1998) investigated the evolution of

virialized and subvirial (thus collapsing) smooth Plummer spheres. They showed that dynamical mass segregation cannot occur rapidly enough in these situations to explain the observations of mass segregation in young (few Myr old) clusters such as the Orion Nebula Cluster (ONC). In this letter we will examine the dynamical evolution of initially cool and clumpy (fractal) stellar distributions. We will show that a combination of subvirial velocities *and* substructure can often lead to dynamical mass segregation on a timescale that is much shorter than would usually be expected. In § 2 we argue that cool, clumpy initial conditions are realistic – indeed, they are both observed and expected from theory. In § 3 we demonstrate that these initial conditions can lead to rapid dynamical mass segregation. In § 4 we discuss the implications of this result and draw our conclusions.

## 2. THE INITIAL CONDITIONS OF STAR CLUSTERS

In this section we review the observations and theory of star cluster formation in molecular clouds. Our aim is to show that both observations and theory lead us to expect that clusters form both subvirial and with substructure. *Star clusters are initially substructured.* Stars form from dense cores in molecular clouds (e.g., Ward-Thompson et al. 2007). Molecular clouds are observed to have significant levels of substructure in both density and kinematics (Williams 1999; Williams et al. 2000; Carpenter & Hodapp 2008). This is not surprising, as we believe that molecular cloud structure is dominated by supersonic turbulence which will produce complex structures (see, Mac Low & Klessen 2004; Ballesteros-Paredes et al. 2007, and references therein). It is not surprising, therefore, that very young ( $< 1$  Myr) star clusters also show significant levels of substructure (Larson 1995; Testi et al. 2000; Elmegreen 2000; Lada & Lada 2003; Gutermuth et al. 2005; Allen et al. 2007). Detailed statistical studies also show the presence of substructure in young clusters, and that substructure is rapidly erased (Cartwright & Whitworth 2004; Schmeja et al. 2008).

*Star clusters form subvirial (cool).* There is increasing evidence that star clusters are born subvirial. Prestellar

<sup>1</sup> Department of Physics and Astronomy, University of Sheffield, Sheffield, S3 7RH, UK; r.allison@sheffield.ac.uk

<sup>2</sup> NAOC-CAS, Beijing, China

<sup>3</sup> Leiden Observatory, P.O. Box 9513, NL-2300 RA Leiden, The Netherlands

cores are often observed to move subsonically (Belloche et al. 2001; André 2002; Walsh et al. 2004; Peretto et al. 2006; Kirk et al. 2007). In hydrodynamic simulations of cluster formation starting from dynamically hot (globally unbound) gas the stars that form also appear to have subvirial motions (see e.g. Bate et al. 2003; Bonnell et al. 2003). Proszkow et al. (2009) also find that the Orion star forming region shows signatures of subvirial dynamics on scales of about 10 pc (see also Adams et al. 2006). Such subvirial motions might well be expected from a turbulent model of star formation in clouds where stars form in converging/colliding flows (see references above, also Adams et al. 2006). It is also worth noting that subvirial initial conditions are required to erase substructure as rapidly as is observed (Goodwin & Whitworth 2004).

### 3. THE EVOLUTION OF COOL, FRACTAL CLUSTERS

In this section we simulate clumpy (fractal), cool clusters and show that they mass segregate dynamically in a time similar to the crossing time of the initial (or final) system.

#### 3.1. Initial Conditions

We simulate the dynamical evolution of star clusters containing 1000 single stars with masses sampled from a three-part power-law mass function (Kroupa 2002), with minimum and maximum masses of  $0.08M_{\odot}$  and  $50M_{\odot}$ , respectively. The stars initially have a fractal distribution within a sphere of radius 1 pc, with initial velocities such that nearby stars have similar velocities, as described in detail by Goodwin & Whitworth (2004). We neglect the effects of stellar evolution because of the short duration of the simulations (4 Myr).

In this letter, we restrict our investigations to clusters with a fractal dimension of 1.6 (giving a very clumpy distribution, where 3.0 gives a uniform sphere) and a virial ratio of  $Q = 0.3$ , where  $Q$  is the ratio of the kinetic to the (modulus of the) potential energy (so that  $Q = 1/2$  is virialized). Such initial conditions give the most extreme dynamical evolution and the most rapid dynamical mass segregation. We will consider a much fuller range of parameter space in a follow-up paper (R. J. Allison et al., in prep.). Here, we just wish to illustrate that rapid dynamical mass segregation can occur with plausible initial conditions, and how this happens. The simulations were carried out using the KIRA integrator in STARLAB. (Portegies Zwart et al. 2001).

#### 3.2. Results

Figures 1(a) and 1(b) show the stellar distributions initially, and after  $\sim 1$  Myr of dynamical evolution, respectively. A comparison of the plots clearly shows that the cluster has evolved from a clumpy and non-mass segregated state to one which has erased substructure and appears to be mass segregated.

We apply the method of Allison et al. (2009), which compares the minimum spanning trees (MSTs) of high-mass stars to those of a random selection of stars to produce a quantitative measure of mass segregation. If the MST of the  $N$  most massive stars is significantly shorter than that of a number of sets of  $N$  random stars then the cluster is mass segregated. The degree of mass segregation can be quantified by the ratio of the lengths of the average randomly selected star MST to the most massive

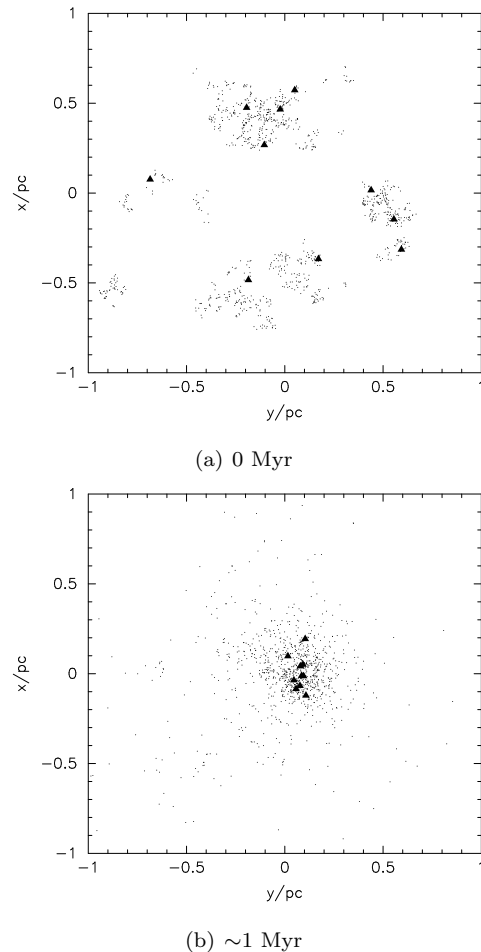


FIG. 1.— Typical stellar distributions at 0 and  $\sim 1$  Myr. The triangles indicate the positions of the 10 most massive stars. In this run the most massive and tenth most massive stars have masses of  $23M_{\odot}$  and  $3.5M_{\odot}$ , respectively.

star MST,  $\Lambda$  (see Allison et al. 2009, for details). The greater  $\Lambda$  is relative to unity, the more mass segregated a cluster is.

Figure 2 shows the evolution of  $\Lambda$  for four subsets of the  $N = 10, 20, 50$  and 100 most massive stars in the cluster. The cluster is not mass segregated initially ( $\Lambda = 1$ ), but after 1 Myr the 10 most massive stars develop a significant level of mass segregation ( $\Lambda \sim 3$ ). The error bars in Figure 2 represent the instantaneous standard deviation at each simulation snapshot.<sup>4</sup> In Figure 2 we can also see that the 20 and 50 most massive stars also mass segregate, but by a much smaller amount. Beyond the 50 most massive stars little mass segregation is seen.

The cluster illustrated in Figure 1 is a fairly typical example of the evolution of cool, highly fractal clusters: a collapse from the cool initial state, which erases substructure and also imprints mass segregation. Unfortunately,

<sup>4</sup> The 10 most massive stars are mass segregated at a similar level for more than a crossing time after the initial violent relaxation phase. In a crossing time the cluster can completely mix and every star in the cluster can migrate to any other position in the cluster. This means that any feature that remains constant over a crossing time is a real feature and if the evolution of the cluster were also accounted for the significance would be much greater than shown.

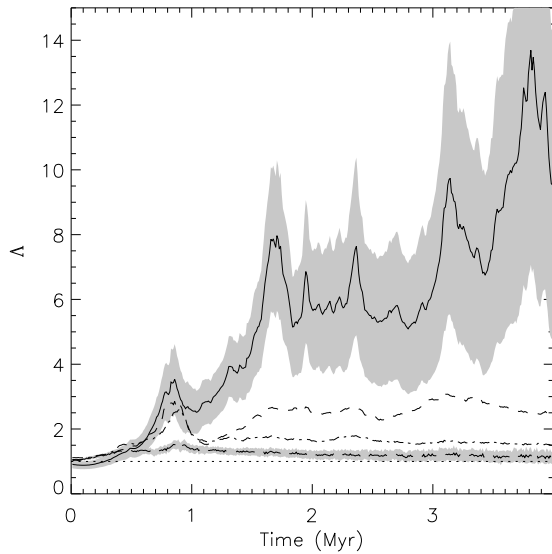


FIG. 2.— The evolution of the mass segregation ratio  $\Lambda$  for the 10, 20, 50 and 100 most massive stars in the cluster (from top to bottom). The envelopes indicate  $1\sigma$  deviations, no error is shown for  $N = 20$  and 50 for clarity. The dotted line indicates a  $\Lambda$  of unity, i.e., no mass segregation.

and unavoidably, using fractal clusters introduces a certain degree of randomness as each fractal – while formally the same (i.e., the same fractal dimension and virial ratio) – is very different in its initial distribution. Thus, when dealing with fractals, it is vital to perform a large ensemble of simulations.

We simulate 50 different clusters, varying only the random number seed used to create the fractal and initial mass function. Of these 50 simulations, 29 mass segregate within 1 Myr, and 44 show mass segregation within 4 Myr. Only 6 show no significant mass segregation by the end of the simulations at 4 Myr. Here, mass segregation is defined as any mass segregation event that lasts for longer than  $\sim 0.1$  Myrs and for which  $\Lambda$  has a significance  $> 1$ .

### 3.3. Mass segregation mechanism

The mass segregation observed in these simulations is from a wholly dynamical origin, and arises from the collapse of the cluster. The initial conditions used in these simulations place the cluster far from equilibrium. The cluster undergoes a gravitational collapse and violent relaxation phase very early on in its evolution in an attempt to virialize itself.

This collapse creates a dense core containing roughly half the mass in a radius of only  $\sim 0.1$  pc. The dense core only lasts for  $0.1 - 0.2$  Myr, but due to its small size, this is around 10 – 20 crossing times of the core, making it dynamically old.

Whilst 10 – 20 crossing times is not enough time to reach full equipartition, it is sufficient time to mass segregate the most massive stars. Spitzer (1969) showed that the mass segregation timescale for a star of mass  $M$ ,  $t_{\text{seg}}(M)$ , is

$$t_{\text{seg}}(M) \approx \frac{\langle m \rangle}{M} t_{\text{relax}}, \quad (1)$$

where  $\langle m \rangle$  is the average mass of a star in the cluster ( $\sim 0.4M_{\odot}$  for a typical initial mass function), and  $t_{\text{relax}}$  is the two-body relaxation timescale of the cluster. The two-body relaxation timescale depends on the number of stars in the cluster,  $N$ , and the crossing time,  $t_{\text{cross}}$ , as

$$t_{\text{relax}} \approx \frac{N}{8 \ln N} t_{\text{cross}}, \quad (2)$$

where the crossing time is simply the radius of the cluster  $R$  divided by the average velocity of a star,  $\sigma$ .

Thus, Eq. 1 can be rewritten as

$$t_{\text{seg}} \approx \frac{\langle m \rangle}{M} \frac{N}{8 \ln N} \frac{R}{\sigma}. \quad (3)$$

Typical values for these parameters for the dense cores are  $N \sim 300 - 500$ ,  $R \sim 0.1 - 0.2$  pc,  $\langle m \rangle = 0.4M_{\odot}$ , and  $\sigma \sim 2$  km s $^{-1}$ . The lifetimes of the dense cores during which they can mass segregate are  $t_{\text{seg}} \sim 0.1$  Myr. This suggests that clusters will be able to mass segregate above  $M \sim 2 - 4M_{\odot}$ .

In Figure 2 we show that below about the 50th most massive star there is no further mass segregation. In this simulation, the mass of the 50th most massive star is  $\sim 2M_{\odot}$  – in good agreement with the calculation presented above.

Interestingly, Allison et al. (2009) find that the ONC appears to be mass segregated down to  $\sim 5M_{\odot}$  but not below that mass. Hillenbrand & Hartmann (1998) also find evidence for mass segregation around  $5M_{\odot}$ , and Moeckel & Bonnell (2009) find that they cannot explain the observations of the ONC unless only the most massive stars are mass segregated. This is exactly the situation predicted in our simulations – mass segregation down to a few solar masses, but not below. Indeed, observations of the mass down to which a cluster is mass segregated (and the dynamical age of the cluster) should provide constraints on the density and duration of the dense phase undergone by the cluster and hence the initial conditions of the cluster.

The mass segregation seen in these simulations is due to the collapse of the cluster and formation of the dense core. Some early, low level, mass segregation is observed in the simulations, due to the dynamical evolution and merger of the sub-clumps in the initial distribution (McMillan et al. 2007), but the presence of long lived, high level mass segregation is due almost entirely to the later evolution of the dense core, not to prior mass segregation in clumps.

The reason that clumpy subvirial clusters are able to mass segregate whilst smooth subvirial clusters do not (such as those simulated by Bonnell & Davies 1998) is due to clumpy clusters being able to collapse to a far denser state than smooth clusters. The potential energy,  $\Omega$ , of a cluster of mass  $M_{\text{clus}}$  and radius  $R$  is given by

$$\Omega = \alpha \frac{GM_{\text{clus}}^2}{R}$$

where  $\alpha$  is a structure parameter (for example in Plummer sphere if  $R$  is the Plummer radius then  $\alpha \sim 0.75$ ).

For a cluster with an initial potential energy  $\Omega_0$  (with radius  $R_0$  and structure parameter  $\alpha_0$ ), and a final potential energy  $\Omega_f$  (with radius  $R_f$  and structure parameter  $\alpha_f$ ) the initial and final potential energies are related to the initial virial ratio  $Q_0$  by

$$\Omega_0(1 - Q_0) = \frac{\Omega_f}{2},$$

assuming the cluster ends virialized. The ratio of initial-to-final radii is

$$\frac{R_0}{R_f} = \frac{\alpha_0}{\alpha_f} 2(1 - Q_0).$$

In the case where a cluster starts smooth (e.g. a Plummer sphere) it will retain this structure and so  $\alpha_0 \sim \alpha_f$  and so if a cluster starts with a virial ratio of  $Q_0 = 0.3$  it will only collapse by a factor of  $\sim 1.4$ . This is not enough to significantly increase the speed of mass segregation. This is consistent with the lack of mass segregation found by Bonnell & Davies (1998) in the collapse of a  $Q_0 = 0.1$  Plummer sphere (which would collapse by a factor of  $\sim 1.8$ ).

However, when a clumpy cluster collapses it erases substructure and becomes smooth. A reasonable estimate for the final structure parameter is that of a Plummer sphere with  $\alpha_f \sim 0.75$ . But the highly fractal initial conditions have a structure parameter which is significantly larger than that for a smooth distribution. For a fractal with fractal dimension = 1.6 numerical experiments show that  $\alpha_0 \approx 1.5$  (this is an approximate figure and can change dramatically depending on the random number seed of the fractal). This means that the degree of collapse to virialise from a realistic initial virial ratio of  $Q_0 = 0.3$  is a factor of about 2.5, from 1pc initially to around 0.4 pc, which is consistent with our simulations (note that the core radius will be significantly smaller than 0.4 pc).

It is important to note that clusters which undergo a collapse and dynamically mass segregate will often be significantly *dynamically* older than their current crossing time might suggest (as also emphasized by Bastian et al. 2008). Clusters can dynamically segregate in 1 – 2 Myr even if this is comparable to their initial or current crossing times (as inferred from their sizes) because they have undergone a dense phase, which means that they are actually many crossing times old (in their cores at least).

The dense cores are very short-lived as the low-mass stars expand due to the transfer of energy from the high-mass stars to low-mass stars during the mass segregation. The lifetimes of the dense phase is usually only 0.1 – 0.2 Myr.

There is one obvious omission from our simulations – gas. For simplicity, we neglect the background potential of the gas from which the star clusters formed. However, we believe that this omission will not affect our results significantly. Much of the gas that does not form stars has a high velocity dispersion: the cloud as a whole may well be unbound due to supersonic turbulence. Stars are

able to form cool as they form in converging and shocked regions within a globally unbound cloud (e.g. Bate et al. 2003; Bonnell et al. 2003). Therefore, we would not expect significant amounts of gas to collapse with the stars (obviously some will, but the contribution to the potential will become small as the stars collapse to a denser configuration). This is observed in the simulations of Bate (2009), where gas which has not formed stars has migrated away from the star forming areas due to its initial velocity dispersion. In particular, Fellhauer, Wilkinson, & Kroupa (2009) show that the presence of a background potential does not significantly alter the merging behavior of subclumps.

#### 4. CONCLUSIONS

Observations and theory suggest that many young star clusters form with a significant amount of substructure. Observations also show that young clusters lose their substructure on a timescale comparable to their current crossing times (1 – 2 Myr). Simulations suggest that the only way in which this could happen is if clusters are born dynamically cool (Goodwin & Whitworth 2004). Therefore we argue that the correct initial conditions of many star clusters must be a cool, clumpy distribution.

We conduct an ensemble of simulations of cool, clumpy clusters (with fractal dimension 1.6 and virial ratio 0.3) to investigate the early dynamical evolution of such clusters. We also find that cool, fractal clusters tend to dynamically mass segregate down to a few solar masses during a dense, but short-lived, state at the end of their collapse. Such limited mass segregation of only the most massive stars is what appears to be observed in the ONC (Allison et al. 2009; Moeckel & Bonnell 2009).

Such rapid dynamical mass segregation in clusters with realistic initial conditions shows that the most massive stars do not have to form in the centers of clusters for very young clusters to be mass segregated. This shows that massive stars could form in relative isolation in large cores and mass segregate later, possibly avoiding the need for competitive accretion to form the most massive stars in the center of a cluster (Bonnell et al. 2001). However, competitive accretion may still play an important role, further increasing the masses of the most massive stars if they mass segregate while gas is still present.

RJA and RJP acknowledge financial support from STFC. SPZ is grateful for the support of the Netherlands Advanced School in Astrophysics (NOVA), the LKBF, and the Netherlands Organization for Scientific Research (NWO). MBNK was supported by PPARC/STFC under grant number PP/D002036/1. This work has made use of the Iceberg computing facility, part of the White Rose Grid computing facilities at the University of Sheffield. We acknowledge the support and hospitality of the International Space Science Institute in Bern, Switzerland, where part of this work was done as part of a International Team Programme.

#### REFERENCES

- Adams F. C., Proszkow E. M., Fatuzzo M., Myers P. C., 2006, *ApJ*, 641, 504
- Allen L., Megeath S. T., Gutermuth R., Myers P. C., Wolk S., Adams F. C., Muzerolle J., Young E., Pipher J. L., 2007, in Reipurth B., Jewitt D., Keil K., eds, *Protostars and Planets V The Structure and Evolution of Young Stellar Clusters*. pp 361–376

- Allison R. J., Goodwin S. P., Parker R. J., Portegies Zwart S. F., de Grijs R., Kouwenhoven M. B. N., 2009, *MNRAS*, 395, 1449
- André, P., 2002, *Ap&SS*, 281, 51
- Ballesteros-Paredes J., Klessen R. S., Mac Low M.-M., Vazquez-Semadeni E., 2007, in Reipurth B., Jewitt D., Keil K., eds, *Protostars and Planets V Molecular Cloud Turbulence and Star Formation*. pp 63–80
- Bastian N., Gieles M., Goodwin S. P., Tranco G., Smith L. J., Konstantopoulos I., Efremov Y., 2008, *MNRAS*, 389, 223
- Bate M. R., 2009, arXiv:0905.3562
- Bate M. R., Bonnell I. A., Bromm V., 2003, *MNRAS*, 339, 577
- Bate M. R., Clarke C. J., McCaughrean M. J., 1998, *MNRAS*, 297, 1163
- Belloche, A., André, P., Motte, F., 2001, in *From Darkness to Light: Origin and Evolution of Young Stellar Clusters*, ASP Conference Series, Vol. 243, eds. Montmerle, T. and André, P., (San Francisco: ASP), p.313
- Bonnell I. A., Bate M. R., 2006, *MNRAS*, 370, 488
- Bonnell I. A., Bate M. R., Vine S. G., 2003, *MNRAS*, 343, 413
- Bonnell, I. A., Clarke, C. J., Bate, M. R., Pringle, J. E., 2001, *MNRAS*, 324, 573
- Bonnell I. A., Bate M. R., Zinnecker H., 1998, *MNRAS*, 298, 93
- Bonnell I. A., Davies M. B., 1998, *MNRAS*, 295, 691
- Carpenter J., Hodapp K., 2008, arXiv:0809.1396
- Cartwright A., Whitworth A. P., 2004, *MNRAS*, 348, 589
- de Grijs R., Johnson R. A., Gilmore G. F., Frayn C. M., 2002a, *MNRAS*, 331, 228
- de Grijs R., Gilmore G. F., Johnson R. A., Mackey A. D., 2002b, *MNRAS*, 331, 245
- de Grijs R., Gilmore G. F., Mackey A. D., Wilkinson M. I., Beaulieu S. F., Johnson R. A., Santiago B. X., 2002c, *MNRAS*, 337, 597
- Elmegreen B. G., 2000, *ApJ*, 530, 277
- Fellhauer M., Wilkinson M. I., Kroupa P., 2009, arXiv:0905.0399
- Goodwin S. P., Bastian N., 2006, *MNRAS*, 373, 752
- Goodwin S. P., Whitworth A. P., 2004, *A&A*, 413, 929
- Gouliermis D., Keller S. C., Kontizas M., Kontizas E., Bellas-Velidis I., 2004, *A&A*, 416, 137
- Gutermuth R. A., Megeath S. T., Pipher J. L., Williams J. P., Allen L. E., Myers P. C., Raines S. N., 2005, *ApJ*, 632, 397
- Hillenbrand L. A., Hartmann L. W., 1998, *ApJ*, 492, 540
- Kirk, H., Johnstone, D., Tafalla, M., 2007, *ApJ*, 668, 1042
- Kraus A. L., Hillenbrand L. A., 2008, *ApJ*, 686, L111
- Kroupa P., 2002, *Science*, 295, 82
- Krumholz M. R., Klein R. I., McKee C. F., 2007, *ApJ*, 656, 959
- Krumholz M. R., McKee C. F., Klein R. I., 2005, *Nature*, 438, 332
- Lada C. J., Lada E. A., 2003, *ARA&A*, 41, 57
- Larson R. B., 1995, *MNRAS*, 272, 213
- Lynden-Bell D., 1967, *MNRAS*, 136, 101
- Mac Low M.-M., Klessen R. S., 2004, *Reviews of Modern Physics*, 76, 125
- McMillan S. L. W., Vesperini E., Portegies Zwart S. F., 2007, *ApJ*, 655, L45
- Moeckel N., Bonnell I. A., 2009, arXiv:0903.3893
- Peretto, N., André, P., Belloche, A., 2006, *A&A*, 445, 979
- Portegies Zwart S. F., McMillan S. L. W., Hut P., Makino J., 2001, *MNRAS*, 321, 199
- Proszkow E. M., Adams F. C., Hartmann L. W., Tobin J. J., 2009, arXiv:0903.3242
- Schmeja S., Klessen R. S., 2006, *A&A*, 449, 151
- Schmeja S., Kumar M. S. N., Ferreira B., 2008, *MNRAS*, 389, 1209
- Spitzer L. J., 1969, *ApJ*, 158, L139
- Testi L., Sargent A. I., Olmi L., Onello J. S., 2000, *ApJ*, 540, L53
- van den Berk J., Portegies Zwart S. F., McMillan S. L. W., 2007, *MNRAS*, 379, 111
- Walsh, A. J., Myers, P. C., Burton, M. G., *ApJ*, 2004, 614, 194
- Ward-Thompson D., André P., Crutcher R., Johnstone D., Onishi T., Wilson C., 2007, in Reipurth B., Jewitt D., Keil K., eds, *Protostars and Planets V An Observational Perspective of Low-Mass Dense Cores II: Evolution Toward the Initial Mass Function*. pp 33–46
- Williams J., 1999, in Franco J., Carraminana A., eds, *Interstellar Turbulence The Structure of Molecular Clouds: are they Fractal?* pp 190
- Williams J. P., Blitz L., McKee C. F., 2000, *Protostars and Planets IV*, pp 97